

# Two Majorana Neutrino Mass Doublets with Thorough Maximal Doublet Mixing from an Analogy with the $K^0$ -Meson Oscillations

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In the recent note [1] we argued that most of the available positive and negative neutrino oscillation data can be incorporated in a four-neutrino weak interaction model with one physical condition: thorough maximal mixing of the neutrino components in each of the two mass doublets. On the level of CP-invariance an extended Pontecorvo analogy between the neutrino oscillations and the  $K^0$ -meson oscillations, which implied maximal violation of the lepton charge conservation in the neutrino mass matrix exclusively, can afford a physical clue to the three data dictated assumptions in a model with three lepton flavors and only two 4-component neutrino generations: four phenomenological neutrinos with two neutrino mass doublets, one sterile neutrino in the oscillations, and thorough maximal neutrino doublet mixing. This analogy predicts that all the four neutrino mass eigenstates are of the Majorana type.

In ref. [1] a simple explicit neutrino mixing and oscillation model with four phenomenological neutrinos [2] – the three weak interaction eigenstates  $\nu_e, \nu_\mu$ , and  $\nu_\tau$  plus one sterile neutrino  $\nu_s$  – is built with one condition: the neutrino mass eigenstates enter the four phenomenological neutrinos only in the form of the eigenstates of the mass doublet neutrino exchange symmetry, symmetric or anti-symmetric. At the level of CP-invariance of the weak interactions this condition is equivalent to the assumption of a thorough maximal mixing of the neutrino mass eigenstates in each of the neutrino mass doublets. The model predicts naturally large oscillation amplitudes for the atmospheric and solar neutrino anomalies and is in good agreement with the positive Super-Kamiokande data [3] if the LSND data [4] are accepted. It is also in agreement with the negative reactor and accelerator data. Though there is yet a long way ahead to the experimental verification of this model, it is certainly interesting that it gets a strong physical support from an extended Pontecorvo analogy between the neutrino oscillations and the well known  $K^0$ -meson oscillations, where the “particle” oscillations had been discovered first. As a useful guiding suggestion, this interesting physical analogy is considered below from the standpoint of a possible clue to the thorough maximal neutrino doublet mixing.

Vacuum neutrino oscillations of the type  $\nu$ -anti- $\nu$  in analogy with the  $K^0$ - $\bar{K}^0$  oscillations were considered in the original Pontecorvo paper [5]. Accordingly, we assume here that the lepton charge is conserved in the weak interactions (as strangeness of the  $K^0$  and  $\bar{K}^0$ -mesons in the strong interactions), but not in the neutrino mass matrix. Its eigenstates can then be the truly neutral (Majorana) neutrino mass states (as the  $K_{1,2}^0$  mesons). In the unreal case with only two lepton flavors  $e$  and  $\mu$ , the minimal model is bounded to one 4-component neutrino  $\nu$  and the definition of the two flavor neutrinos is

$$\nu_e = \nu = \frac{1}{\sqrt{2}}(\nu_1 + \nu_2), \quad \nu_\mu = \tilde{\nu} = \frac{1}{\sqrt{2}}(\nu_1 - \nu_2), \quad (1)$$

with  $\nu_1$  and  $\nu_2$  being the two neutrino Majorana mass eigenstates,

$$\nu_1 = \frac{1}{\sqrt{2}}(\nu + \tilde{\nu}), \quad \nu_2 = \frac{1}{\sqrt{2}}(\nu - \tilde{\nu}), \quad (2)$$

in strict analogy with the definition of the  $K_{1,2}^0$  meson states with definite combined CP-parities. All four neutrino states participate in the weak interactions here, and there is no sterile neutrino. The definition (1) means that the electron and muon neutrinos have opposite lepton charges, but it does not preclude the strict conservation of the lepton charge in the weak interactions. The transitions  $(\nu_e)_L \leftrightarrow (\tilde{\nu}_\mu)_R$  and  $(\nu_\mu)_L \leftrightarrow (\tilde{\nu}_e)_R$  are induced by the neutrino masses, and their probabilities are very small for relativistic neutrinos. The only way to the experimental detection of the truly neutral neutrino constituent states  $\nu_1$  and  $\nu_2$  within the weak interaction eigenstates in Eq. (1) is through the observation of the neutrino oscillations ( $\nu_e \leftrightarrow \nu_\mu$ ;  $\nu_e \rightarrow \nu_e$ ;  $\nu_\mu \rightarrow \nu_\mu$ ) with maximal oscillation amplitudes – in contrast to the  $K^0$ -meson case, where the  $K_{1,2}^0$  constituents of the kaons had been discovered first by

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their decay modes, and the oscillations had been observed later. In the realistic case with three lepton flavors  $e, \mu$  and  $\tau$ , the minimal model is bounded to two 4-component neutrinos  $\nu_I$  and  $\nu_{II}$ . The necessity of a second 4-component neutrino with the same lepton charge is a data dictated condition – by the lepton flavor proliferation and the LSND indication. The most general mixing pattern of these two neutrinos and their anti-particles  $\tilde{\nu}_I$  and  $\tilde{\nu}_{II}$ , with the substantial condition of strict conservation of the lepton charge in the short-range weak interactions, is a  $4 \times 4$  unitary mixing matrix with two parameters which does not mix the neutrino particle and anti-particle states. It determines the following four 4-component phenomenological neutrino states:

$$\nu_e = \nu_I \cos \theta + \nu_{II} \sin \theta, \quad (3)$$

$$\nu_\mu = -\nu_I \sin \theta + \nu_{II} \cos \theta, \quad (4)$$

$$\nu_\tau = \tilde{\nu}_I \sin \phi + \tilde{\nu}_{II} \cos \phi, \quad (5)$$

$$\nu_s = \tilde{\nu}_I \cos \phi - \tilde{\nu}_{II} \sin \phi. \quad (6)$$

The weak interaction eigenstates are  $(\nu_e)_L$ ,  $(\nu_\mu)_L$  and  $(\nu_\tau)_L$ ; the appearance of one 2-component sterile neutrino  $(\nu_s)_L$  is an inevitable result here. Only eight of the sixteen neutrino components in the Eqs. (3)–(6) are independent: the right neutrinos  $(\nu_e)_R$  and  $(\nu_\mu)_R$  (and the corresponding left anti-neutrinos) are linear superpositions of the right anti-neutrinos  $(\tilde{\nu}_\tau)_R$  and  $(\tilde{\nu}_s)_R$  (and the corresponding left neutrinos), and vice-versa (there are only two independent 4-component neutrino generations, e.g.  $\nu_e$  and  $\nu_\mu$  and in the particular case with  $\phi = -\theta$  it would be  $\nu_\tau = \tilde{\nu}_\mu$ ,  $\nu_s = \tilde{\nu}_e$ ). It means that the three lepton numbers  $n_e$ ,  $n_\mu$ , and  $n_\tau$  are not conserved in the local weak interactions if we take into account the effects of the neutrino masses. Only one lepton charge can be strictly conserved in the local weak interactions of the massive neutrinos, it is the one carried by the 4-component neutrinos  $\nu_I$  and  $\nu_{II}$ . The condition in the Eqs. (3) and (4) that the  $\nu_e$  and  $\nu_\mu$  share the same two neutrino states is an indication which follows from the LSND data [4]. The formulation of the neutrino mixing model (3)–(6) with two 4-component neutrinos is unique because of its apparent discrete exchange symmetry  $\nu_I \leftrightarrow \tilde{\nu}_I$ ,  $\nu_{II} \leftrightarrow \tilde{\nu}_{II}$  and the above mentioned condition of strict conservation of the lepton charge in the weak interactions, plus the indication from the LSND effect. The only two free parameters here, the two mixing angles  $\theta$  and  $\phi$ , have to be determined from the experimental data (the parameter  $\theta$  is the LSND mixing angle). The neutrino pairs  $(\nu_e, \nu_\mu)$  and  $(\nu_\tau, \nu_s)$  have opposite lepton charges with a definition of the “leptons”:  $\nu_e, \nu_\mu, \tilde{\nu}_\tau, \tilde{\nu}_s, e^-, \mu^-$ , and  $\tau^-$ ; and the “anti-leptons”:  $\tilde{\nu}_e, \tilde{\nu}_\mu, \nu_\tau, \nu_s, e^+, \mu^+$ , and  $\tau^+$ . The probabilities of the transitions  $(\nu_e)_L, (\nu_\mu)_L \leftrightarrow (\tilde{\nu}_\tau)_R, (\tilde{\nu}_s)_R$  and  $(\tilde{\nu}_e)_R, (\tilde{\nu}_\mu)_R \leftrightarrow (\nu_\tau)_L, (\nu_s)_L$ , induced by the neutrino masses, are very small for relativistic neutrinos with  $m_\nu^2/E_\nu^2 \ll 1$ .

Above we considered the two 4-component neutrinos  $\nu_I$  and  $\nu_{II}$  as regular Dirac particles with masses  $m_1$  and  $m_2$ . Only at this level is the lepton charge strictly conserved in the weak interactions of massive neutrinos. In accordance with the  $\nu\bar{\nu}K^0$  analogy we assume now that the lepton charge is not conserved in the neutrino mass matrix, and its eigenstates  $\nu_i$  and  $\nu'_i$ ,  $i=1,2$ , are the truly neutral neutrino mass states:

$$\begin{aligned} \nu_I &= \frac{1}{\sqrt{2}}(\nu_1 + \nu'_1), & \tilde{\nu}_I &= \frac{1}{\sqrt{2}}(\nu_1 - \nu'_1), \\ \nu_{II} &= \frac{1}{\sqrt{2}}(\nu_2 + \nu'_2), & \tilde{\nu}_{II} &= \frac{1}{\sqrt{2}}(\nu_2 - \nu'_2). \end{aligned} \quad (7)$$

With the statement in Eq. (7) the neutrino mixing model (3)–(6) has the special feature of “thorough maximal neutrino doublet mixing” which is discussed in ref. [1]; the correspondence with the notations in this reference is  $\nu_{1,2}^s = \nu_{I,II}$ ,  $\nu_{1,2}^a = \tilde{\nu}_{I,II}$ , i.e. the two 4-component neutrinos  $\nu_{I,II}$  and their anti-particles  $\tilde{\nu}_{I,II}$  in the Eqs. (3)–(6) are the four eigenstates  $\nu_i^s$  and  $\nu_i^a$ ,  $i=1,2$ , of the exchange symmetry operators of the neutrino Majorana mass eigenstates  $\nu_i$  and  $\nu'_i$  in the two mass doublets. Thus we can suggest that the “thorough maximal neutrino doublet mixing” of ref. [1] is due to the physical conditions of lepton charge conservation in the weak interactions and that the neutrino mass eigenstates are of the Majorana type, with definite combined CP-parities; the thorough maximal neutrino doublet mixing in the weak interactions is a manifestation of the implied maximal violation of the lepton charge conservation in the neutrino mass matrix, which is the sole effective location of lepton nonconservation in the present model. Note that the implied here characteristics of the neutrino mass matrix with regard to the discrete symmetries are opposite to the ones of the neutrino weak interactions: the neutrino mass matrix violates maximally the lepton charge conservation, but does not mix the components of the two neutrino generations  $\nu_I$  and  $\nu_{II}$ . Therefore, the lepton charge nonconservation in the weak interactions of the massive Majorana neutrinos is induced only by the neutrino mass doublet splittings and should be very small in comparison with the aforementioned lepton number nonconservations in the important case of very narrow neutrino mass doublets. Since no neutrino decay modes are known, the only way to the experimental detection of the Majorana mass constituent states in the weak interaction

neutrino eigenstates is by the observations of the neutrino oscillations with maximal amplitudes. Thus the discovery of large neutrino mixing in the Super-Kamiokande oscillation experiment [3] can be regarded as an indication in this direction.

The thorough maximal neutrino doublet mixing in the model (3)–(7) leads to a number of distinct and characteristic predictions [1]. The data of the solar, atmospheric and the LSND experiments can be explained here with the following values of the neutrino mass squared differences [2],

$$\begin{aligned}\Delta m_1^2 &\equiv \Delta m_{solar}^2 \sim 10^{-10} (Vac), \text{ or } \sim 10^{-5} (MSW) \text{ eV}^2, \\ \Delta m_2^2 &\equiv \Delta m_{atm}^2 \sim 10^{-3} - 10^{-2} \text{ eV}^2, \\ \Delta m_{12}^2 &\equiv \Delta m_{LSND}^2 \sim 1 \text{ eV}^2,\end{aligned}\tag{8}$$

and a possible scheme of the neutrino mass spectrum

$$\underbrace{\overbrace{m_1 < m'_1}^{solar} \ll \overbrace{m_2 < m'_2}^{atm}}_{LSND}.\tag{9}$$

In the other possible scheme the positions of the “solar” and the “atm” mass splittings are exchanged. The long-baseline oscillations in the mixing model (3)–(7), (8) and (9), including the atmospheric and solar ones with maximal oscillation amplitudes, are lepton charge neutrino-antineutrino  $\nu_{I,II} \leftrightarrow \bar{\nu}_{I,II}$  oscillations (as the oscillations of strangeness in the  $K^0$  case), whereas the short-baseline ones are neutrino generation  $\nu_I \leftrightarrow \nu_{II}$  and  $\bar{\nu}_I \leftrightarrow \bar{\nu}_{II}$  oscillations (with no  $K^0$  analogy: they would remain unchanged even if the neutrino mass matrix were of the regular Dirac type with no neutrino doublet splittings). The equations below are explicitly written in terms of the first scheme, the conclusions are valid for both of them. The probability of the appearance  $\nu_\mu \leftrightarrow \nu_e$  oscillations:

$$|\langle \nu_\mu(0) | \nu_e(L) \rangle|^2 = \sin^2 2\theta \left[ \left\langle \sin^2 \left( \frac{\Delta m_{12}^2 L}{4E} \right) \right\rangle - \frac{1}{4} \sin^2 \left( \frac{\Delta m_1^2 L}{4E} \right) - \frac{1}{4} \sin^2 \left( \frac{\Delta m_2^2 L}{4E} \right) \right],\tag{10}$$

where  $\theta = \theta_{LSND}$ , and the symbol  $\langle \rangle$  in the first term denotes the arithmetic mean value of the appropriate four factors related to the four “large” mass squared differences among the two neutrino mass doublets. The notations are

$$\Delta m_{1,2}^2 = m_{1',2'}^2 - m_{1,2}^2, \quad \Delta m_{12}^2 \cong m_2^2 - m_1^2,\tag{11}$$

$E$  is the initial beam energy and  $L$  is the distance from the source. From the Eq. (10), the  $\nu_\mu \leftrightarrow \nu_e$  oscillations, both the short-baseline and the long-baseline, are determined by one factor  $\sin^2 2\theta$ , which is the LSND oscillation amplitude with the estimation [4]  $\sin^2 2\theta \approx 3 \times 10^{-3}$ .

The probability of the  $\nu_\mu \rightarrow (\nu_\tau + \nu_s)$  appearance oscillations,

$$W(\nu_\mu \rightarrow \nu_\tau + \nu_s) = \cos^2 \theta \sin^2 \left( \frac{\Delta m_2^2 L}{4E} \right) + \sin^2 \theta \sin^2 \left( \frac{\Delta m_1^2 L}{4E} \right),\tag{12}$$

is independent of the second mixing angle  $\phi$  and also of the neutrino mass doublet separation  $\Delta m_{12}^2$  and, therefore, describes only long-baseline oscillations, atmospheric and possibly terrestrial. The first term in this equation is the dominant one, it describes the main part of the atmospheric  $\nu_\mu$  oscillations with the amplitude

$$A_{atm} \cong \cos^2 \theta \cong 1.\tag{13}$$

The probability of the coming long-baseline accelerator  $\nu_\mu$  survival oscillations can be described with a good approximation by the single term

$$W(\nu_\mu \rightarrow \nu_\mu) \cong 1 - W(\nu_\mu \rightarrow \nu_\tau + \nu_s) \cong \cos^2 \left( \frac{\Delta m_2^2 L}{4E} \right),\tag{14}$$

with  $L$  the distance from the  $\nu_\mu$  source to the detector.

The probability of the  $\nu_e \rightarrow (\nu_\tau + \nu_s)$  appearance oscillations is:

$$W(\nu_e \rightarrow \nu_\tau + \nu_s) = \cos^2 \theta \sin^2 \left( \frac{\Delta m_1^2 L}{4E} \right) + \sin^2 \theta \sin^2 \left( \frac{\Delta m_2^2 L}{4E} \right).\tag{15}$$

The first term in this equation is the dominant one and describes the solar neutrino appearance oscillations with the amplitude

$$A_{solar} \cong \cos^2 \theta \cong 1. \quad (16)$$

The probability of the solar vacuum neutrino survival oscillations,

$$W(\nu_e \rightarrow \nu_e) \cong 1 - W(\nu_e \rightarrow \nu_\tau + \nu_s) \cong \cos^2 \left( \frac{\Delta m_1^2 L_I}{4E} \right), \quad (17)$$

is just the original still viable Pontecorvo [5] 2-neutrino maximal mixing solution for the solar neutrino deficit (here  $L_I$  is the Earth-Sun distance).

The probability of the  $\nu_\tau \rightarrow (\nu_\mu + \nu_e)$  appearance oscillations,

$$W(\nu_\tau \rightarrow \nu_\mu + \nu_e) = \cos^2 \phi \sin^2 \left( \frac{\Delta m_2^2 L}{4E} \right) + \sin^2 \phi \sin^2 \left( \frac{\Delta m_1^2 L}{4E} \right), \quad (18)$$

is independent of the first mixing angle  $\theta$ , and of  $\Delta m_{12}^2$ , and suggests that there are only long-baseline oscillations of this kind.

The probability of the  $\nu_\tau \rightarrow \nu_s$  oscillations is described by the expression at the right side of the Eq. (10) with  $\theta \rightarrow \phi$ . From the big-bang nucleosynthesis implications [6] probably it follows

$$\sin^2 2\phi < 10^{-7}. \quad (19)$$

Such a constraint implies (even if the astrophysical estimations would weaken it by several orders of magnitude) that from the continuum of possibilities in the atmospheric  $\nu_\mu \rightarrow (\nu_\tau + \nu_s)$  and solar  $\nu_e \rightarrow (\nu_\tau + \nu_s)$  oscillation dominances only the two extremes ( $\nu_\mu \rightarrow \nu_\tau; \nu_e \rightarrow \nu_s$ ) or ( $\nu_\mu \rightarrow \nu_s; \nu_e \rightarrow \nu_\tau$ ) are allowed.

The main conclusions are: 1). In the present model, the basic description of the neutrinos in the weak interactions is by charge carrying 4-component spinors as in all the other known cases of the fermions. The peculiarity of the neutrino case is that here it is rather enough to have only two 4-component neutrino generations, instead of three 4-component generations used for the charged leptons and quarks. These two neutrino generations are reduced to four 2-component Majorana neutrino mass eigenstates because of the maximal parity nonconservation in the weak interactions and the maximal lepton charge nonconservation in the neutrino mass matrix. The complete superposition set of the left elements of these four Majorana neutrinos is structuring the complete set of the left components of the four phenomenological neutrinos  $(\nu_e)_L, (\nu_\mu)_L, (\nu_\tau)_L$  and  $(\nu_s)_L$  in the Eqs. (3)–(6), and this structure determines the different types of neutrino oscillations; 2). The atmospheric  $\nu_\mu$  and the solar  $\nu_e$  oscillation amplitudes are naturally large if the LSND data are accepted; 3). The transitions  $\nu_\mu \rightarrow \nu_\tau + \nu_s$  and  $\nu_e \rightarrow \nu_\tau + \nu_s$  give the main contributions to the atmospheric and the solar neutrino oscillations because of the small LSND mixing angle  $\theta$ . The dependence on the second mixing angle  $\phi$  leads to the different “complementarity” relations between the separate  $\nu_\tau$  and  $\nu_s$  channels in the sum  $(\nu_\tau + \nu_s)$ ; 4). The contribution of the  $\nu_\mu \leftrightarrow \nu_e$  transitions to both of these oscillations are smaller by more than two orders of magnitude. The last three inferences are in good agreement with the positive Super-Kamiokande data [3]; 5). The four phenomenological neutrinos are divided into two different pairs  $(\nu_e, \nu_\mu)$  and  $(\nu_\tau, \nu_s)$  in conformity with the two different pairs of the particle and anti-particle states of the 4-component neutrinos  $(\nu_I, \nu_{II})$  and  $(\tilde{\nu}_I, \tilde{\nu}_{II})$ . This feature of the model leads to the prediction of negative results for the  $\nu_\mu, \nu_e \rightarrow \nu_\tau, \nu_s$  oscillation searches in the short-baseline accelerator experiments such as CHORUS and NOMAD [7], and of large oscillation amplitudes at the appropriate long-baseline experiments, in contrast to the  $\nu_\mu \leftrightarrow \nu_e$  and, probably,  $\nu_\tau \leftrightarrow \nu_s$  oscillations; 6). The deviation from unity of the  $\nu_e \rightarrow \nu_e$  survival probability is small,  $\leq \sin^2 2\theta_{LSND}$  for oscillation distances  $L \ll 2\pi E / \Delta m_1^2$ . This is in good agreement with the  $\nu_e$  disappearance oscillation data such as Bugey [8], and the appearance oscillation  $\nu_\mu \rightarrow \nu_e$  data from the BNL E776 experiment [9], and also with the long-baseline reactor  $\nu_e$  disappearance CHOOZ experiment [10] where this restriction on the distances is fulfilled quite well; 7). Though all the neutrino mass eigenstates are of the Majorana nature, the double  $\beta$  decay in the model is extremely suppressed by the factors of the neutrino doublet splittings with the dominant  $\Delta m_1^2$  from the Eq. (8), because of the condition of lepton charge conservation in the weak interactions of unsplit doublet neutrinos. The allowed by the lepton charge conservation reactions  $\mu^- + A(Z) \rightarrow A(Z-2) + \tau^+$  and also decay modes  $\tau^+ \rightarrow \mu^- + \pi^+ + \pi^+$ , etc., are mediated by the neutrino masses, but the latter introduce small factors.

The predictions above appear to be straightforward inferences of the analogy between the neutrino and the  $K^0$ -meson oscillation phenomena in the minimal description with only two 4-component neutrino generations and three

lepton flavors. It is a remarkable fact that this simple and interesting physical analogy is in good agreement with the majority of the available positive and negative oscillation data. Note, however, that the acceptance of the LSND data [4] is crucial for this agreement.

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